

Lesson 6-6 Homework Answer Key

Day 1

① $D = 2x - x^2$
 $D' = 2 - 2x$
 $0 = 2 - 2x$
 $2x = 2$
 $x = 1$

$2-2x$	+		-
		1	
		MAX	

② $S = x + x^2$
 $S' = 1 + 2x$
 $0 = 1 + 2x$
 $-1 = 2x$

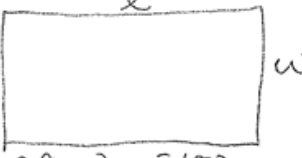
$1+2x$	-		+
		$-\frac{1}{2}$	
		MIN	

③ $D = x - x^2$
 $D' = 1 - 2x$
 $0 = 1 - 2x$
 $1 = 2x$
 $x = \frac{1}{2}$

$1-2x$	+		-
		$\frac{1}{2}$	
		MAX	

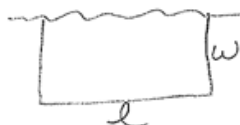
④ $x - y = 10 \rightarrow x = 10 + y$
 $P = xy$
 $= (10 + y)(y)$
 $= 10y + y^2$
 $P' = 10 + 2y$
 $0 = 10 + 2y$
 $-2y = 10$
 $y = -5$
 $x = 5$

$10+2y$	-		+
		-5	
		MIN	

⑤ 

$2l + 2w = 100$
 $l + w = 50$
 $w = 50 - l$
 $A = lw$
 $A = l(50 - l)$
 $= 50l - l^2$
 $A' = 50 - 2l$
 $0 = 50 - 2l$
 $l = 25 \quad w = 25$

$50-2l$	+		-
		25	
		MAX	

⑥ 

$l + 2w = 100 \rightarrow l = 100 - 2w$
 $A = lw$
 $= (100 - 2w)w$
 $= 100w - 2w^2$
 $A' = 100 - 4w$
 $0 = 100 - 4w$
 $w = 25 \quad l = 100 - 2 \cdot 25 = 50$

$100-4w$	+		-
		25	
		MAX	

Day 2

① $D = x^2 - 13$
 $D' = 2x - 3x^2$
 $0 = x(2 - 3x)$
 $x = 0$ $x = \frac{2}{3}$

x	-	1	+	+	
$2-3x$	+	1	+	1	-
D'	-	0	+	$\frac{2}{3}$	-
			MIN		MAX

$x = \frac{2}{3}$

② $x+y=20 \rightarrow x=20-y$
 $p = xy^2$
 $p = (20-y)y^2$
 $= 20y^2 - y^3$
 $0 = 40y - 3y^2$
 $0 = y(40 - 3y)$
 $y = 0$ $y = \frac{40}{3}$

y	-	1	+	+	
$40-3y$	+	1	+	1	-
p'	-	0	+	$\frac{40}{3}$	-
			MIN		MAX

$y = \frac{40}{3}$ $x = \frac{20}{3}$

③ $xy = 64 \rightarrow x = \frac{64}{y}$
 $S = x + y$
 $S = \frac{64}{y} + y = 64y^{-1} + y$
 $S' = -64y^{-2} + 1$
 $0 = -64y^{-2} + 1$
 $0 = 1 - 64y^{-2}$
 $= (1 - 8y^{-1})(1 + 8y^{-1})$
 $y = 8$ $y = -8$

$1+\frac{8}{y}$	+	1	-	1	+	+
$1-\frac{8}{y}$	+	1	+	1	-	+
S'	+	-8	-	0	-	8
						MIN

$x = 8$ $y = 8$ $x = -\frac{1}{4}$

④ $S = x + 2x^2$
 $S' = 1 + 4x$
 $0 = 1 + 4x$
 $x = -\frac{1}{4}$

$1+4x$	-	1	+	
S'	-	1	+	$-\frac{1}{4}$
				MIN

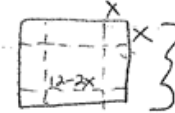
⑤ $x - y = 22$
 $x = 22 + y$
 $p = xy$
 $= (22 + y)y$
 $= 22y + y^2$
 $p' = 22 + 2y$
 $0 = 22 + 2y$
 $y = -11$
 $x = 11, y = -11$

$22+2y$	-	1	+	
p'	-	22	+	-11
				MIN

⑥ $x^3 - 3x^2 = D$
 $3x^2 - 6x = D'$
 $3x(x - 2) = 0$
 $x = 0$ $x = 2$

$3x$	-	1	+	1	+
$x-2$	-	1	-	2	+
D'	+	0	-	2	+
					MIN

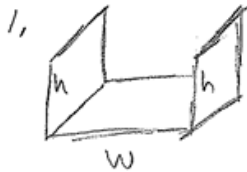
$x = 2$

⑦  12
 $V = lwh$
 $= (12 - 2x)^2(x)$
 $= (144 - 48x + 4x^2)x$
 $= 144x - 48x^2 + 4x^3$
 $V' = 144 - 96x + 12x^2$
 $0 = 12(x^2 - 8x + 12)$
 $= 12(x - 6)(x - 2)$
 $x = 6$ $x = 2$

$12x^2$	-	1	+	1	+
x^2	-	1	+	1	+
V'	+	144	-	96	+
					MAX

$2'' \times 8'' \times 8''$

Day 3



$$2h + w = 12 \rightarrow w = 12 - 2h$$

$$A = wh$$

$$A = h(12 - 2h)$$

$$= 12h - 2h^2$$

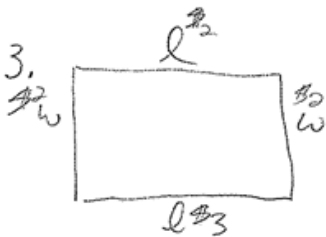
$$A' = 12 - 4h$$

$$0 = 12 - 4h$$

$$h = 3$$

$$h = 3'', w = 6''$$

	+	-
12	4	
3		
		MAX



$$\text{Cost: } 2l + 2w + 2w + 3l = 100$$

$$4w + 5l = 120$$

$$l = 24 - \frac{4}{5}w$$

$$A = lw$$

$$= (24 - \frac{4}{5}w)w$$

$$= 24w - \frac{4}{5}w^2$$

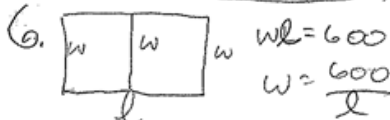
$$A' = 24 - \frac{8}{5}w$$

$$0 = 24 - \frac{8}{5}w$$

$$w = 15$$

$$w = 15', l = 12'; \text{ 83\% side} = 12'$$

	+	-
24	8	
15		
		MAX



$$p = 3w + 2l$$

$$= 3(\frac{600}{l}) + 2l$$

$$= \frac{1800}{l} + 2l = 1800l^{-1} + 2l$$

$$p' = -1800l^{-2} + 2$$

$$0 = \frac{-1800}{l^2} + 2$$

$$-2 = \frac{-1800}{l^2}$$

$$l = \pm 30$$

	+	-
-1800	2	
30		
		MIN
		MAX

$$l = 30'$$

$$w = 20'$$



$$r^2 + h^2 = (6\sqrt{3})^2$$

$$r^2 + h^2 = 108$$

$$r^2 = 108 - h^2$$

$$V = \frac{1}{3}Bh$$

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi (108 - h^2)h$$

$$= 36\pi h - \frac{1}{3}\pi h^3$$

$$V' = 36\pi - \pi h^2$$

$$0 = \pi(36 - h^2)$$

$$0 = \pi(6+h)(6-h)$$

$$h = -6 \quad h = 6$$

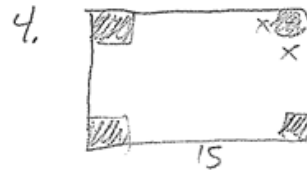
$$r^2 = 108 - 36$$

$$= 72$$

$$r = \pm \sqrt{72}$$

	+	-
36	1	
6		
		MAX

$$h = 6'' \quad r = \sqrt{72}''$$



$$V = (8-2x)(15-2x)x$$

$$= 120x - 46x^2 + 4x^3$$

$$V' = 120 - 92x + 12x^2$$

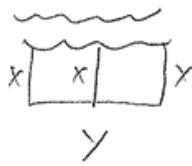
$$0 = 4(3x-5)(x-6)$$

$$x = \frac{5}{3} \quad x = 6$$

	+	-
120	92	
12		
		MIN
		MAX

$$\frac{5}{3}'' \times \frac{14}{3}'' \times \frac{35}{3}''$$

5.



$$3x + y = 180$$

$$y = 180 - 3x$$

$$A = xy$$

$$= x(180 - 3x)$$

$$= 180x - 3x^2$$

$$A' = 180 - 6x$$

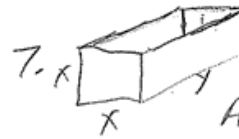
$$0 = 180 - 6x$$

$$x = 30$$

$$\begin{array}{r} 180 - 6x \\ + \quad | \quad - \\ \hline 30 \\ \text{MAX} \end{array}$$

$$\boxed{x = 30'$$

$$y = 90'}$$



$$x^2 y = 36 \rightarrow y = \frac{36}{x^2}$$

$$A = 2x^2 + 3xy$$

$$= 2x^2 + 3x\left(\frac{36}{x^2}\right)$$

$$= 2x^2 + \frac{108}{x}$$

$$= 2x^2 + 108x^{-1}$$

$$A' = 4x - 108x^{-2}$$

$$0 = 4x - 108x^{-2}$$

$$\frac{108}{x^2} = 4x \rightarrow 4x^3 - 108 = 0$$

$$4(x^3 - 27) = 0$$

$$x = 3$$

$$\begin{array}{r} x^3 - 27 \\ - \quad | \quad + \\ \hline 3 \\ \text{MIN} \end{array}$$

$$\boxed{w = h = 3''}$$

$$l = 4''}$$

8.



$$\pi r^2 h = 8\pi$$

$$r^2 h = 8$$

$$h = \frac{8}{r^2}$$

$$A = \pi r^2 + 2\pi r h$$

$$= \pi r^2 + 2\pi r \left(\frac{8}{r^2}\right)$$

$$= \pi r^2 + \frac{16\pi}{r}$$

$$= \pi r^2 + 16\pi r^{-1}$$

$$A' = 2\pi r - 16\pi r^{-2}$$

$$0 = 2\pi r - 16\pi r^{-2}$$

$$2\pi r = 16\pi r^{-2}$$

$$r = 8r^{-2}$$

$$r^3 = 8$$

$$r = 2$$

$$\begin{array}{r} r^3 - 8 \\ - \quad | \quad + \\ \hline 2 \\ \text{MIN} \end{array}$$

$$\boxed{r = 2'', h = 2''}$$

9.



$$r^2 + h^2 = (10\sqrt{3})^2$$

$$r^2 = 300 - h^2$$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi (300 - h^2) h$$

$$= 100\pi h - \frac{1}{3}\pi h^3$$

$$V' = 100\pi - \pi h^2$$

$$0 = 100\pi - \pi h^2$$

$$\cancel{\pi} h^2 = 100\cancel{\pi}$$

$$h = \pm 10$$

$$\begin{array}{r} 100 - h^2 \\ - \quad | \quad + \quad | \quad - \\ \hline -10 \quad 10 \\ \text{MAX} \end{array}$$

$$h = 10'', r = \sqrt{200}''$$

Day 3

10.



$$r+h=15$$

$$h=15-r$$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi r^2 (15-r)$$

$$= 5\pi r^3 - \frac{1}{3}\pi r^3$$

$$V' = 10\pi r - \pi r^2$$

$$0 = 10\pi r - \pi r^2$$

$$= r(10-r)$$

$$r=0 \quad r=10$$

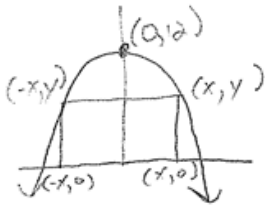
$$\begin{array}{cccc} r & - & 1 & + & 1 & + \\ 10r & + & 1 & + & 1 & - \\ \hline & - & 0 & + & 10 & - \\ & & & & \text{MAX} & \end{array}$$

$$r = 10''$$

$$h = 5''$$

$$\text{Max volume: } \frac{500\pi}{3} \text{ in}^3$$

12. $y = 12 - x^2$



$$A = lw$$

$$= 2x(12 - x^2)$$

$$= 24x - 2x^3$$

$$A' = 24 - 6x^2$$

$$0 = 24 - 6x^2$$

$$0 = 6(4 - x^2)$$

$$0 = (2+x)(2-x)$$

$$x = -2 \quad x = 2$$

$$\begin{array}{cccc} 2x & - & 1 & + & 1 & + \\ 2x & + & 1 & + & 1 & - \\ \hline A' & - & 2 & + & 2 & - \\ & & & & \text{MAX} & \end{array}$$

$$x = 2, y = 8$$

$$\text{Max Area} = 32$$

11.



$$2r+2h=24$$

$$r+h=12$$

$$h=12-r$$

$$V = \pi r^2 h$$

$$= \pi r^2 (12-r)$$

$$= 12\pi r^2 - \pi r^3$$

$$V' = 24\pi r - 3\pi r^2$$

$$0 = 24\pi r - 3\pi r^2$$

$$\begin{array}{cccc} r & - & 1 & + & 1 & + \\ 24r & + & 1 & + & 1 & - \\ \hline & - & 0 & + & 8 & - \\ & & & & \text{MAX} & \end{array}$$

$$r=0 \quad r=8$$

$$r = 8'' \quad h = 4''$$

13.



$$V = \pi r^2 h = 8000\pi$$

$$r^2 h = 8000$$

$$h = \frac{8000}{r^2}$$

$$A = \pi r^2 + 2\pi r h$$

$$= \pi r^2 + 2\pi r \left(\frac{8000}{r^2}\right)$$

$$= \pi r^2 + \frac{16000\pi}{r}$$

$$= \pi r^2 + 16000\pi r^{-1}$$

$$A' = 2\pi r - 16000\pi r^{-2}$$

$$0 = 2\pi r - 16000\pi r^{-2}$$

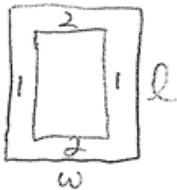
$$2\pi r = \frac{16000\pi}{r^2}$$

$$r^3 = 8000$$

$$r = 20$$

$$r = 20\text{ft} \quad h = 20\text{ft}$$

14.



$$lw = 100$$

$$w = \frac{100}{l}$$

$$A = (l-2)(w-4)$$

$$= (l-2)\left(\frac{100}{l} - 4\right)$$

$$= 100 - 4l - \frac{200}{l} + 8$$

$$= 108 - 4l - 200l^{-1}$$

$$A' = -4 + 200l^{-2}$$

$$0 = -4 + 200l^{-2}$$

$$4 = \frac{200}{l^2}$$

$$4l^2 = 200 \rightarrow l^2 = 50$$

$$l = \pm\sqrt{50}$$

$$\begin{array}{cccc} & - & 1 & + & 1 & + \\ & & & & & & \\ & & & & & & \\ \hline A' & - & \sqrt{50} & + & \sqrt{50} & - \\ & & & & \text{MAX} & \end{array}$$

$$l = \sqrt{50}'' = 5\sqrt{2}''$$

$$w = \frac{100}{\sqrt{50}}'' = 10\sqrt{2}''$$

15.



$$S.A. = 2\pi r^2 + 2\pi rh = 96\pi$$

$$2r^2 + rh = 48$$

$$r^2 + rh = 48$$

$$h = \frac{48 - r^2}{r}$$

$$V = \pi r^2 h$$

$$= \pi r^2 \left(\frac{48 - r^2}{r} \right)$$

$$= 48\pi r - \pi r^3$$

$$V' = 48\pi - 3\pi r^2$$

$$= 3\pi (16 - r^2)$$

$$= 3\pi (4+r)(4-r)$$

$$\downarrow \quad \downarrow$$

$$r = -4 \quad r = 4$$

$$\boxed{r = 4 \text{ in } h = 8 \text{ in}}$$

$$\begin{array}{cccc} 4r & - & 1 & + & 1 & + \\ 4-r & + & 1 & + & 1 & - \\ \hline & - & 4 & + & 4 & - \\ & & & & & \text{MAX} \end{array}$$

16.



$$V = w^2 h = 375$$

$$h = \frac{375}{w^2}$$

$$Cost = 3 \cdot 2w^2 + 1 \cdot 4wh$$

$$(cents) = 6w^2 + 4wh$$

$$= 6w^2 + 4w \left(\frac{375}{w^2} \right)$$

$$= 6w^2 + 1500w^{-1}$$

$$C' = 12w - 1500w^{-2}$$

$$0 = 12w - 1500w^{-2}$$

$$12w = \frac{1500}{w^2}$$

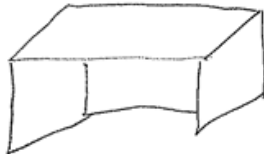
$$12w^3 - 1500 = 0$$

$$12(w^3 - 125) = 0$$

$$\boxed{w = 5 \text{ in } h = 15 \text{ in}}$$

$$\begin{array}{ccc} w^3 - 125 & - & 1 & + \\ & & 5 & \\ \hline & & & \text{MIN} \end{array}$$

17.



$$SA = 2h^2 + 2wh = 96$$

$$h^2 + wh = 48$$

$$w = \frac{48 - h^2}{h}$$

$$V = h^2 w$$

$$= h^2 \left(\frac{48 - h^2}{h} \right)$$

$$= 48h - h^3$$

$$V' = 48 - 3h^2$$

$$0 = 48 - 3h^2$$

$$0 = 3(16 - h^2)$$

$$0 = 3(4+h)(4-h)$$

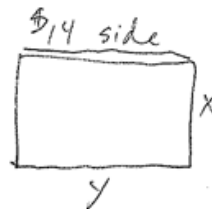
$$\downarrow \quad \downarrow$$

$$h = -4 \quad h = 4$$

$$\begin{array}{cccc} 4+h & - & 1 & + & 1 & + \\ 4-h & + & 1 & + & 1 & - \\ \hline & - & 4 & + & 4 & - \\ & & & & & \text{MAX} \end{array}$$

$$\boxed{h = 4 \text{ ft}} \\ \boxed{w = 8 \text{ ft}}$$

18.



$$xy = 600$$

$$y = \frac{600}{x}$$

$$Cost = 7(2x+y) + 14y$$

$$= 14x + 7\left(\frac{600}{x}\right) + 14\left(\frac{600}{x}\right)$$

$$= 14x + \frac{12600}{x}$$

$$= 14x + 12600x^{-1}$$

$$C' = 14 - 12600x^{-2}$$

$$0 = 14 - 12600x^{-2}$$

$$= 14x^2 - 12600$$

$$= 14(x^2 - 900)$$

$$= 14(x+30)(x-30)$$

$$\begin{array}{ccc} x+30 & - & 1 & + & 1 & + \\ x-30 & - & 1 & - & 1 & + \\ \hline & + & - & 30 & - & 30 & + \\ & & & & & & \text{MIN} \end{array}$$

$$\boxed{x = 30 \text{ ft } y = 20 \text{ ft}}$$